coupling constants are small. We assert that this smallness is no less mysterious than the smallness of $1-\alpha_P(0)$, and we suggest that the two mysteries may be related.

## ACKNOWLEDGMENTS

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# High-Energy Photoproduction of $\rho^0$ and $\phi$ Mesons: A Composite View of Vector Mesons*

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We construct a model of high-energy $\rho^0$- and $\phi$-meson photoproduction in which the incident photon produces a charged pion (kaon) pair near the target. The photon coupling to the meson pair is just the electric charge $e$. A virtual meson undergoes diffraction scattering from the target and the $\rho^0(\phi)$ is seen as a final-state interaction of the meson pair. There are no free parameters in the model. Agreement with presently existing high-energy experiments is quite good.

## I. INTRODUCTION

THERE now exists persuasive evidence in support of the vector dominance viewpoint[1] toward photon interactions with hadrons. The essential feature of this viewpoint is the supposition that the interacting photon behaves as though it contains a coherent mixture of all nonstrange, vector, isosinglet, and isovector mesons. The hadronic interactions of the photon then occur by means of the strong interactions of the photon's own hadronic content. As a consequence, the interactions of photons with hadronic matter, especially at high energy, are economically parametrized in terms of experimentally determined vector-meson–photon coupling constants and independently measured (in principle) strong interaction amplitudes.

Despite its successes, however, it seems to us that the vector dominance viewpoint should not be an exclusive one. For one thing, it is an essentially phenomenological construct, and it might be possible to gain additional insight (and prediction) from an alternative and more detailed way of looking at the same phenomena. Also, on the basis of esthetics, at least, one might raise the objection that the vector mesons in a free (zero mass) photon are far from their "mass shell," and the connection between the strong interactions of virtual and "physical" vector mesons is by no means obvious. Thus, the question of "measurability" of the vector meson interactions may require some clarification.

The $\rho$-photoproduction process is a particularly convenient one for examining the consequences of an alternative point of view. As a matter of observation, the physical $\rho$ meson is simply a highly correlated system of two pions.[2] As one moves the energy "off shell" (which is to say, when one considers the two-pion system at energies different from that corresponding to the $\rho$ peak) the degree of correlation is reduced, as is indicated by the behavior of the $p$-wave scattering amplitude. Near the two-pion scattering threshold, in fact, a pair of $p$-wave pions is essentially uncorrelated, and it seems somewhat presumptuous to speak of a $\rho$ meson (as distinct from a pion pair) in this energy region. Thus, with reference to a massless photon it should be at least as meaningful to speak of its "two-pion content" as its "$\rho$ content." In this way we are led to consideration of the general problem of producing pion pairs[3] by high-energy photons with small momentum transfer to the target. Thus, the essential ingredients of the model are the photon coupling to the charged pion pair, pion-nucleon scattering at high energy and low momentum transfer, and the $p$-wave pion-pion interaction.

We consider, then, photoproduction of a pion pair from a proton calculated according to the diagrams of Fig. 1(a). The $\pi^+\pi^-$ pair must, of course, be emitted by the photon in a relative $p$ wave to conserve angular momentum. If the pion-nucleon scattering which "realizes" the virtual pion is strongly diffractive, as would be expected at high energy, then the scattering

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[1] S. C. C. Ting, Rapporteur's Summary, in *Proceedings of the XIV International Conference on High Energy Physics at Vienna, September 1968* (CERN, Geneva, 1968), p. 43; M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); M. Gell-Mann and F. Zachariasen, *ibid.* **124**, 953 (1961); M. Ross and L. Stodolsky, *ibid.* **149**, 1172 (1966).
[2] M. Gell-Mann and F. Zachariasen, Ref. 1.
[3] S. D. Drell, Phys. Rev. Letters **5**, 278 (1960); P. Söding, Phys. Letters **19**, 702 (1966); A. S. Krass, Phys. Rev. **159**, 1496 (1967).

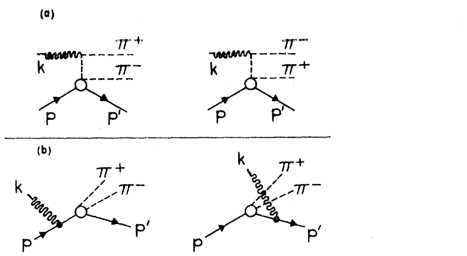

FIG. 1. Pion pair production from a proton. (a) The direct process in which one pion undergoes diffraction scattering from the proton. (b) Graphs added to make the amplitude gauge-invariant.

will be mostly with small momentum transfer to the proton. But at small momentum transfer, the relative angular momenta of the two pions is hardly changed at all so that the two final pions will be mostly in a relative $p$ state. In fact, for exactly forward scattering the pion pair must have exactly the angular momentum and "helicity" of the incident photon.

It is instructive, now, to compare the calculation just described with the vector dominance calculation depicted in Fig. 2. The final $\rho$ meson must again, as a consequence of angular momentum conservation, carry the incident photon helicity when the production is exactly forward. However, for production in other than the forward direction, the helicity of the final $\rho$ is determined by the dynamics of the scattering process. Within the framework of vector dominance, no prediction can be made. From the pion pair picture, on the other hand, one obtains a detailed account of the final $\pi$-$\pi$ state as a function of the momentum transfer to the proton. In other words, the dynamics required in the vector dominance picture is provided by the two-pion picture. Furthermore in this framework, we also have a model for the background effects in $\rho$-meson photoproduction. The background amplitude just consists of the pion-pairs in all other angular momentum states except the $p$ wave. Consequently, every angular momentum state of the two-pion system is represented by only one amplitude, whereas in earlier work on the interference effects in $\rho$-meson photoproduction[4] the $p$-wave amplitude was represented by the sum of the diagrams in Figs. 1 and 2. In our view this is "double counting."

It is clear that everything said in this introduction about $\rho$ photoproduction through intermediate $\pi^+\pi^-$ pairs is equally valid for $\phi$ photoproduction through intermediate $K^+K^-$ pairs. In the following, we shall formulate and discuss the model for $\rho$ photoproduction

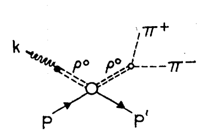

FIG. 2. Conventional vector-dominance mechanism.

[4] P. Söding, Ref. 3; A. S. Krass, Ref. 3.

with the understanding that our results stand also for $\phi$ production if the appropriate changes of mass and coupling parameters are made. It also might be possible to develop a model for $\omega$ photoproduction along similar lines by considering $\omega$ as a resonance in the three pion system. Since this would be a nontrivial extension of the two-pion model, it will not be considered in this paper.

In the next section we provide a brief reminder of the vector dominance calculation of $\rho^0$ photoproduction as proposed by Ross and Stodolsky[1] and augmented by the quark model predictions of Joos and of Kajantie and Trefil.[5] Section III is devoted to some kinematical and notational preparation for our discussion of the two-pion model which is begun in Sec. IV. That section ends with a comparison of the predictions of the two models for the production of "stable" (narrow-width) $\rho^0$'s. Section V is devoted to a discussion of the differences between the two models. In Sec. VI we compare the predictions of the two-pion model for $\rho$ production and the two-kaon model for $\phi$ production with experiment. Finally, Sec. VII contains our conclusions.

## II. VECTOR DOMINANCE AND QUARKS

Following Gell-Mann,[1] we write the coupling constant of the neutral $\rho$ meson to the electromagnetic current as

$$g_{\gamma\rho}=em_\rho^2(2\gamma_\rho)^{-1}. \tag{1}$$

Then for production of a narrow $\rho$ by diffraction dissociation, we have

$$\frac{d\sigma}{dt}(\gamma p\to\rho^0 p)=\tfrac{1}{4}\alpha\frac{4\pi}{\gamma_\rho^2}\frac{d\sigma}{dt}(\rho^0 p\to\rho^0 p)\,, \tag{2}$$

where $(d\sigma/dt)(\rho^0 p\to\rho^0 p)$ is the cross section for the scattering of transverse $\rho^0$'s by protons and $\alpha$ the fine-structure constant. Assuming that the $\rho^0 p$ scattering amplitude is purely imaginary and making use of the optical theorem leads to the result that at zero-momentum transfer

$$\frac{d\sigma}{dt}(\gamma p\to\rho^0 p)\big|_{t=0}=\tfrac{1}{4}\alpha\frac{4\pi}{\gamma_\rho^2}\frac{\sigma_T^2(\rho^0 p)}{16\pi}\,. \tag{3}$$

From the quark model one predicts[5] that the total cross section for $\rho_0 p$ scattering obeys the relation

$$\sigma_T(\rho^0 p)=\tfrac{1}{2}[\sigma_T(\pi^+ p)+\sigma_T(\pi^- p)]\,, \tag{4}$$

where $\sigma_T(\pi^\pm p)$ is the total cross section for $\pi^\pm$ on protons. Further, if we assume that the $\rho$ meson dominates the pion electromagnetic form factor, then there is the further prediction[1]

$$\gamma_\rho=\tfrac{1}{2}g_{\rho\pi\pi}\,, \tag{5}$$

[5] H. Joos, Phys. Letters **24B**, 103 (1967); K. Kajantie and J. S. Trefil, *ibid.* **24B**, 106 (1967). See also X. Bucella and M. Collocci, *ibid.* **24B**, 16 (1967).

where $g_{\rho\pi\pi}$ is related to the $\rho$ width by the relation

$$\Gamma(\rho\to\pi\pi)=\frac{1}{12}\frac{g_{\rho\pi\pi}^2}{4\pi}\left[1-\left(\frac{2m_\pi}{m_\rho}\right)^2\right]^{3/2}. \qquad (6)$$

Then Eq. (3) becomes

$$\frac{d\sigma}{dt}(\gamma p\to\rho^0 p)\big|_{t=0}=\alpha\frac{4\pi}{g_{\rho\pi\pi}^2}\frac{[\sigma_T(\pi^+p)+\sigma_T(\pi^-p)]^2}{64\pi}, \qquad (7)$$

which is essentially the prediction of Ref. 5. One might also suppose that the momentum transfer dependence in Eq. (2) should also be the same as that for $\pi^\pm p$ elastic scattering, at least at very high energies. If this is true, then the expected distribution[6] would be approximately $e^{at}$, where $t$ is the squared four-momentum transfer, and $a$ is known to be about 8 $\text{GeV}^{-2}$.

The discussion given so far is applicable to a $\rho$ meson of infinitely narrow width. Ross and Stodolsky,[7] in their discussion, find that the mass distribution is given, in the vicinity of the resonance, by[8]

$$\left(\frac{m_\rho}{m}\right)^4\frac{1}{\pi}\frac{\frac{1}{2}\Gamma}{(m-m_\rho)^2+\frac{1}{4}\Gamma^2}dm. \qquad (8)$$

## III. KINEMATICAL AND NOTATIONAL PRELIMINARIES

We calculate the cross section for a photon and proton of 4-momenta $k$ and $p$, respectively, to collide and produce a charged pion pair and proton with respective momenta $q_+$, $q_-$, and $p'$. We denote the total invariant mass of the two pion system by

$$m^2=-(q_++q_-)^2=(q_++q_-)_0^2-(\mathbf{q}_++\mathbf{q}_-)^2, \qquad (9)$$

which also specifies our metric.

If the squared 4-momentum transfer to the proton is

$$t=-(p-p')^2 \qquad (10)$$

then the energy of the photon in the rest system of the pion pair $(\mathbf{q}_++\mathbf{q}_-=0)$ is

$$k_0=|\mathbf{k}|=(2m)^{-1}(m^2-t) \qquad (11)$$

and the velocity of one of the pions in this system is $(\hbar=c=1)$

$$\beta=[1-(2m_\pi/m)^2]^{1/2}. \qquad (12)$$

In this same system we shall choose coordinates so that the $z$ axis is along $\mathbf{k}$ and $\mathbf{p}$ is in the $x$-$z$ frame. Then some invariants we will need are

$$-k\cdot q_\pm=\tfrac{1}{4}(m^2-t)(1\mp\beta\cos\theta), \qquad (13a)$$

$$-k\cdot p=\tfrac{1}{2}(s-M^2)\approx\tfrac{1}{2}s, \qquad (13b)$$

$$-k\cdot p'=\tfrac{1}{2}(s+t-M^2-m^2)\approx\tfrac{1}{2}s, \qquad (13c)$$

where $M$ is the proton mass, $s$ is the square of the total barycentric energy (assumed very large), and we consistently drop terms of order $s^{-1}$. Making the corresponding high-energy approximation leads to the result that the cosine of the angle between $\hat{k}$ and $\hat{p}$ is

$$\xi=\hat{k}\cdot\hat{p}=-(m^2+t)/(m^2-t). \qquad (14)$$

We shall work in the Coulomb gauge in the two-pion rest system. Then if $\boldsymbol{\varepsilon}^\pm$ is the photon polarization vector for $\pm$ polarization, we find that ($\epsilon^\pm$ is the polarization 4-vector)

$$\epsilon^\pm\cdot q_+=-\epsilon^\pm q\cdot_-=\mp 2^{-1/2}(e^{\pm i\phi}\sin\theta)\times\tfrac{1}{2}\beta m, \qquad (15a)$$

$$\epsilon^\pm\cdot p=\epsilon^\pm\cdot p'=\mp 2^{-1/2}[ms/(m^2-t)](-t/m^2)^{1/2}. \qquad (15b)$$

Let us denote by $s_\mp$ the square of the total energy of the (virtual) $\pi^\pm$-proton collision in its own barycentric system. Then

$$s_\mp=-(p+k-q_\mp)^2\approx\tfrac{1}{2}s(1\pm\beta\cos\chi)+O(M^2), \qquad (16a)$$

where

$$\cos\chi=\hat{q}_+\cdot\hat{p}=-\left(\frac{m^2+t}{m^2-t}\right)\cos\theta+\frac{2m^2}{m^2-t}\times(-t/m^2)^{1/2}\sin\theta\cos\phi. \qquad (16b)$$

It is of some interest to examine the mass of the virtual pion taking part in the collision. For the $\pi^+$, for example, this is

$$\mu_v^2=(k-q_-)^2=m_\pi^2-\tfrac{1}{2}(m^2-t)(1-\beta\cos\theta). \qquad (17)$$

For zero-momentum transfer and $m$ at the mass of the $\rho$, the range is about

$$0\geqslant\mu_v^2\geqslant-0.6, \qquad (18)$$

in units of $\text{GeV}^2$. Thus, we see that the virtual pion is close to its mass shell only when it is forward scattered. In the backward direction, the mass extrapolation is about the same as that for the $\rho$ in a vector-dominance calculation.

We shall work in terms of a transition matrix $\mathfrak{M}^\pm$, which is related to the cross section by

$$\sigma=\tfrac{1}{2}(2\pi)^{-5}\frac{M^2}{s}\int d^3p'd^3q_+'d^3q_-'(4p_0'\omega_+\omega_-)^{-1}\times\delta^{(4)}(P_f-P_i)\sum_{j=\pm}|\mathfrak{M}^j|^2$$

$$=\tfrac{1}{2}(4\pi)^{-4}\left(\frac{M}{s}\right)^2\int dm^2dtd\Omega_+\,\beta\sum_{j=\pm}|\mathfrak{M}^j|^2, \qquad (19)$$

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[6] See Ting, Ref. 1, for example. However, we are not aware that this supposition has ever been justified on theoretical grounds.

[7] M. Ross and L. Stodolsky, Ref. 1.

[8] In our opinion the inclusion of a $(m_\rho/m)^4$ factor in the $\rho$ shape is not justified within their model.

where $d\Omega_+$ is the differential solid angle of the outgoing $\pi^+$ in the two-pion rest system, and one-half of the indicated sum represents the polarization average for the incident photon.

Finally, we shall denote by $T(\pm)$ the scattering amplitude for the $\pi^\pm$ on the proton, and assume that it is permissible to represent the amplitude as though it were dominated completely by diffractive scattering so that

$$T(\pm)=iAs_\mp e^{at}, \qquad (20)$$

where $A$ is purely real and is related to the total cross section by

$$A=(2M)^{-1}\sigma_T(\pi p)$$
$$=(4M)^{-1}[\sigma_T(\pi^+p)+\sigma_T(\pi^-p)]. \qquad (21)$$

## IV. ρ PRODUCTION AS A DIPION PROCESS

The matrix element corresponding to the process in Fig. 1 is[3]

$$\mathfrak{M}_0^\pm=-e\left[\frac{\epsilon^\pm\cdot q_+}{k\cdot q_+}T(-)-\frac{\epsilon^\pm\cdot q_-}{k\cdot q_-}T(+)\right]$$
$$=-ieA\left[\left(\frac{\epsilon^\pm\cdot q_+}{k\cdot q_+}-\frac{\epsilon^\pm\cdot q_-}{k\cdot q_-}\right)\frac{s_++s_-}{2}+\left(\frac{\epsilon^\pm\cdot q_+}{k\cdot q_+}+\frac{\epsilon^\pm\cdot q_-}{k\cdot q_-}\right)\frac{s_+-s_-}{2}\right]e^{at/2}. \qquad (22)$$

In writing Eq. (22) we have made use of the fact that the pion form factor for coupling to a real photon is just the charge $e$ and that off-mass shell effects can be neglected except in the pole of the pion propagator. It is evident that the term proportional to $(s_++s_-)$ in Eq. (22) is gauge-invariant, whereas the term proportional to $s_+-s_-$ is not. We shall force gauge invariance by adding nucleon pole terms to $\mathfrak{M}^\pm$ [see Fig. 1(b)]. Although this is not a unique prescription, we believe that it should not introduce gross errors. Therefore, we add to $\mathfrak{M}_0^\pm$ the term

$$\mathfrak{M}_1^\pm=-ieA\left(\frac{\epsilon^\pm\cdot p}{kp}+\frac{\epsilon^\pm\cdot p'}{kp'}\right)\frac{s_+-s_-}{2}e^{at/2} \qquad (23)$$

so that the total matrix element is

$$\mathfrak{M}^\pm=\mathfrak{M}_0^\pm+\mathfrak{M}_1^\pm. \qquad (24)$$

It will be found that the contribution of $\mathfrak{M}_1^\pm$ to the matrix element is small, and vanishes for $t=0$. Thus, the effect of including the term $\mathfrak{M}_1^\pm$ suggests that our treatment of gauge invariance does not lead to the omission of important terms.

With the help of Sec. III it is now a trivial matter to see that $\mathfrak{M}^\pm$ may be put into the form

$$\mathfrak{M}^\pm=-2^{-1/2}eAs2\beta m(m^2-t)^{-1}\mathfrak{A}^\pm e^{at/2}, \qquad (25)$$

where

$$\mathfrak{A}^\pm=\mp[(1-\beta^2\cos^2\theta)^{-1}\sin\theta e^{\pm i\phi}(1+\beta^2\cos\theta\cos\chi)-(-t/m^2)\cos\chi]. \qquad (26)$$

We are especially interested in the $p$-wave projections of $\mathfrak{A}^\pm$ which we denote by $\mathfrak{A}_1^\pm$. These are readily found to be

$$\mathfrak{A}_1^+=(1/1+y)\{[(1-2y+2yf)e^{i\phi}-ye^{-i\phi}]\sin\theta-y^{1/2}(1+y-2f)\cos\theta\}, \qquad (27a)$$

$$\mathfrak{A}_1^-=(1+y)^{-1}\{[(1-2y+2yf)e^{-i\phi}-ye^{i\phi}]\sin\theta-y^{1/2}(1+y-2f)\cos\theta\}, \qquad (27b)$$

where

$$f(\beta^2)=\frac{3}{2\beta^2}\left[1-(2\beta)^{-1}(1-\beta^2)\ln\frac{1+\beta}{1-\beta}\right] \qquad (28)$$

and

$$y=-t/m^2. \qquad (29)$$

We note that at $t=0$ the amplitude $\mathfrak{A}^\pm$ is purely $p$ wave and transverse, just as advertised in the introduction. Also, for nonzero values of $t$ $\mathfrak{A}^t$ contains only odd angular momenta of the dipion system as befits a pion pair with unit isospin.

In order to discuss $\rho$-meson photoproduction, it is now required that we put the $\rho$ meson, which is to say, the final-state $\pi$-$\pi$ scattering, into the theory. We write that

$$\mathfrak{A}_\rho^\pm(m^2,\theta,\phi)=(\mathfrak{A}^\pm-\mathfrak{A}_1^\pm)+\mathfrak{A}_1^\pm F(m^2), \qquad (30)$$

where the first term represents the "background," $F(m^2)$ represents the final-state interaction, and the problem now is to determine $F(m^2)$.

Conventional discussions of final-state interactions start with the assumption that enhancement factors such as $F(m^2)$ must carry the phase of the elastic scattering of the final-state strongly interacting particles (the $\pi$-$\pi$ $p$-wave phase shift, in our case). This can be proved if the production vertex is weak or electromagnetic[9] or in certain potential models. One may then construct simple models of the production process that can be solved to find the enhancement factor.[10] It is characteristic of such solutions that they involve knowledge of the elastic scattering at all physically accessible energies. It follows that unless there is a complete theory of scattering and production available such "solutions" are of limited value for computational purposes.[11]

In the absence of a convincing theory we have chosen to take a more phenomenological approach. The idea is to take for $|F(m^2)|$ the enhancement factor that is

[9] K. Watson, Phys. Rev. **88**, 1163 (1952); R. D. Amado, *ibid.* **158**, 1414 (1967); I. J. R. Aitchison and C. Kacser, *ibid.* **173**, 1700 (1968).

[10] For a review see John R. Gillespie, *Final State Interactions* (Holden-Day, Inc., San Francisco, 1964).

[11] The approach expressed here is our interpretation of some remarks made to us by David Horn (private communication).

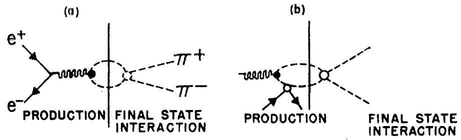


FIG. 3. Determination of the final-state interaction. In (a) and (b) we identify the portions of the two graphs to the right of the vertical line. The photon coupling in (a) is just the charge $e$.

measured in a different process (see Fig. 3) that we believe is related to the one considered here. At the same time, we shall conform with popular dogma by assuming that $F(m^2)$ does have the phase of elastic $\pi$-$\pi$ scattering at total barycentric energy $m$.

We consider for this purpose the process

$$e^+ + e^- \to \rho^0 \to \pi^+ + \pi^- \tag{31}$$

for which measurements now exist.[1] We think of this process being dominated by the graph in Fig. 3(a), where the virtual photon interaction with the dipion is just the charge $e$ (i.e., the form factor is unity), while the structure is a consequence of the final-state interaction depicted by the "blob" on the right of the figure. The enhancement factor for this process was fit (within experimental error) by Gounaris and Sakurai[12] with an expression that may be written as

$$\begin{aligned} F(m^2) &= [m_\rho/\Gamma_\rho g(m)] e^{i\delta(m^2)} \sin\delta(m^2) \\ &\approx m_\rho^2 [m^2 - m_\rho^2 - im\Gamma_\rho g(m)]^{-1}, \end{aligned} \tag{32}$$

where the second expression may be used in the vicinity of the $\rho$ mass,

$$g(m) = [\beta(m)/\beta(m_\rho)]^3 (m/m_\rho)^2 \tag{33}$$

and $\delta(m^2)$ is just the $\pi$-$\pi$ $p$-wave elastic scattering phase shift.

Our viewpoint is that the final state $\pi$-$\pi$ interactions in the two processes, $e^+ + e^- \to \rho^0$ and $\rho^0$ photoproduction, should be the same. That is, we are identifying the two "blobs" on the right of the Figs. 3(a) and 3(b) so that photoproduction is "the same as" production by annihilation except for the insertion of an additional "external" interaction on an intermediate pion line. This assumption then permits us to use the enhancement factor of Eq. (32) in Eq. (30).

The cross section for dipion production is now found to be

$$\begin{aligned} \frac{d^4\sigma}{dt\,dm^2 d\Omega_+} &= \alpha(4\pi)^{-3} m^{-2} (1+y)^{-2} \beta^3 [|\mathcal{A}^+|^2 + |\mathcal{A}^-|^2] \\ &\quad \times \tfrac{1}{16} [\sigma_T(\pi^+ p) + \sigma_T(\pi^- p)]^2 e^{at}. \end{aligned} \tag{34}$$

For present purposes we consider only the $p$-wave contribution to this expression ($\rho^0$ production) and find that in the narrow-width approximation

$$\begin{aligned} \frac{d\sigma}{dt}(\gamma p \to \rho^0 p) &= 4\alpha \frac{4\pi}{g_{\pi\pi}^2} \frac{1}{64\pi} \\ &\quad \times [\sigma_T(\pi^+ p) + \sigma_T(\pi^- p)]^2 H(y_\rho) e^{at}, \end{aligned} \tag{35}$$

where

$$\begin{aligned} H(y_\rho) = \{ &(1+y)^{-4} [(1-2y+2yf)^2 + y^2 \\ &+ \tfrac{1}{2} y(1+y-2f)^2] \}_{y=y_\rho} \end{aligned} \tag{36}$$

and the subscript means that the dipion mass is given the value at the $\rho$ peak. One should notice that $H(0)$ is unity so that the forward cross section in Eq. (35) is exactly four times that given by the vector-dominance calculation in Sec. II.

## V. COMPARISON WITH VECTOR-DOMINANCE MODEL

It is difficult to make a meaningful comparison of our last result Eq. (35) with the vector-meson dominance and simple quark-model prediction. But it can be seen quite clearly where the difference of the two models comes from. This is conveniently discussed in terms of the quark content in the photon interaction. In the two-pion dissociation model developed in Sec. IV, the photon dissociates into two quark-antiquark ($\bar{q}q$) pairs, whereas in the model discussed in Sec. II the photon dissociates into only one $\bar{q}q$ pair. This difference in introducing the coupling of the photon or virtual $\rho$-meson, two $\bar{q}q$ pairs versus one $\bar{q}q$ pair, is essentially responsible for the fact that the two models differ by a factor of 2 in their amplitudes. It can be seen more easily if we assume that the quark mass were low enough to produce $\rho$ mesons in $\bar{q}q$ scattering. Then we can apply the same dynamical model as in Sec. IV but now starting from the process of producing $q\bar{q}$ pairs on protons by photons. Obviously, we then would arrive at the same result as Eq. (35) but without the factor of 4 since at the lower vertex only the $q$-$p$ or $\bar{q}$-$p$ amplitudes, which are half the $\pi^\pm$-$p$ amplitudes, appear. Under such circumstances we must come to the conclusion that the $\pi^+$-$\pi^-$ dissociation model presented here and the naive $q\bar{q}$-dissociation model are not compatible, and must differ by a factor of 4 in their predictions simply as a matter of quark counting.

What, then, is the relationship between the two models? In the absence of a reliable method for treating processes with virtual hadrons, we do not really know. However, we find it tempting to suppose that the two models represent two different ways of "sweeping under the rug" the off-the-mass-shell effects. If this supposition is correct, then the quarks should probably be regarded as mathematical objects which undergo only the simplest type of scattering process. That is to say, the $\rho^0$-nucleon scattering is to be regarded as the coherent superposition of two quark-nucleon single scatterings without need for corrections due to off-the-

[12] G. J. Gounaris and J. J. Sakurai, Phys. Rev. Letters **21**, 244 (1968). See also M. T. Vaughn and K. C. Wali, *ibid.* **21**, 938 (1968), for a similar model.

mass-shell effects or multiple scatterings. Then, by implication, the intermediate pion approach would require substantial correction for off-the-mass-shell effects in order that the two models be consistent and provided that the quark model gives the best representation of physical reality. On the other hand, if our model gives the better representation of physics, then the simple quark model requires substantial correction and the quark picture becomes somewhat gratuitous.

## VI. COMPARISON WITH EXPERIMENT

### A. $\rho^0$ Photoproduction

The theoretical model that we have constructed is meant to be applicable in the case of asymptotically large energies and small values of $-t$. Therefore, in evaluating Eq. (35) we have used 20 mb for the value of the $\pi^{\pm}$-$p$ total cross section[13] and taken for the $t$ dependence the form[6] $e^{at+bt^2}$ with $a=9.2$ GeV$^{-2}$ and $b=2.6$ GeV$^{-4}$. The prediction is plotted in Fig. 4, where it is compared with the recently published data of Jones *et al.*,[14] Davier *et al.*,[15] and McClellan *et al.*[16]

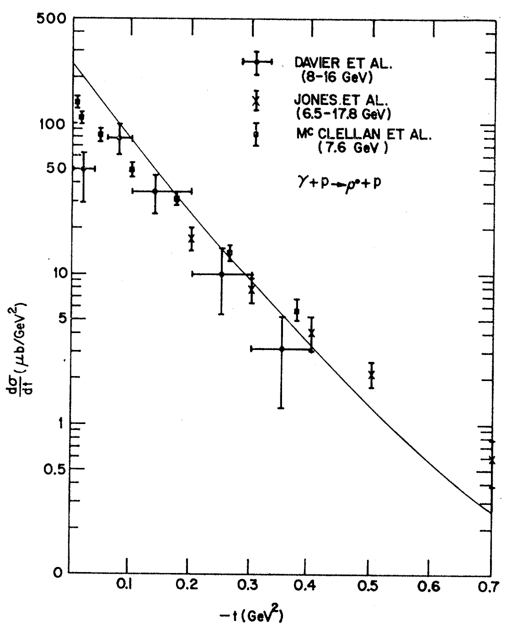


FIG. 4. Differential cross section for photoproduction of the $\rho^0$ meson versus $t$, the square of the four-momentum transfer to the proton. The solid curve is the absolute prediction of the theory. The experimental data are from Refs. 14–16. The apparent decrease of the experimental cross section near $-t=0$ may be spurious.

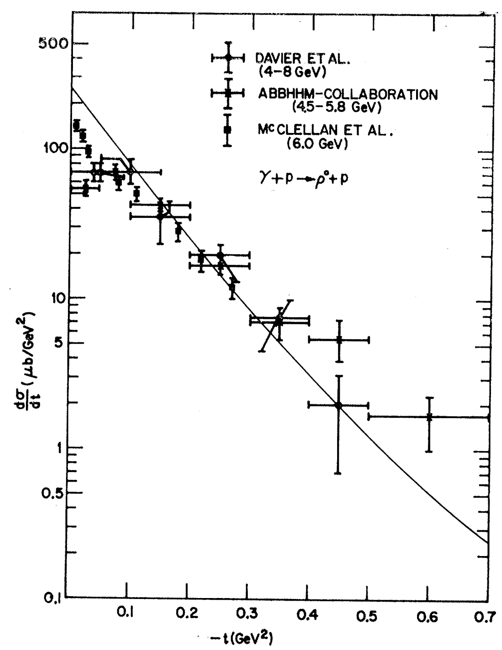


FIG. 5. Differential cross section for photoproduction of the $\rho^0$ meson versus $t$, the square of the four-momentum transfer to the proton. The solid curve is the absolute prediction of the theory. The experimental data for energies around 6 GeV are from Refs. 15–17.

In Fig. 4 we have taken only the data obtained at the energies above 6 GeV. A comparison with data at lower energies[15–17] is shown in Fig. 5. In our opinion, the comparison is satisfactory for $-t$ values less than about 0.5 GeV$^2$. We understand that the apparent decrease of the experimental cross section of Refs. 15 and 17 for $-t\lesssim 0.05$ GeV$^2$ is likely to be a spurious effect caused by experimental biases.

Over the range shown, the $t$ dependence of the curves in Figs. 4 and 5 is given very nearly by

$$d\sigma/dt \sim [1-(t/m_\rho^2)]^{-2} e^{at+bt^2}. \quad (37)$$

The first factor in this expression is proportional to the square of the photon energy evaluated in the $\rho$ rest frame. It comes from the $k^{-1}$ factor in the amplitude that is expected in the description of the radiation or absorption of a photon by a charged particle. Consequently, this factor is inherent in the model and accounts for the increase in the slope of $d\sigma/dt$ beyond that observed in pion-nucleon scattering. We also mention that because of the factor $(1-t/m^2)^{-2}$, the $t$ dependence is a function of the mass $m$ of the two-pion system. As a result, we are able to make the qualitative prediction that the effective slope of the $t$ distribution is

[13] See Y. Sumi, Progr. Theoret. Phys. (Kyoto) Suppl. **41–42**, 3 (1967), for a compilation of elastic meson-nucleon elastic scattering data. For asymptotic cross sections see M. Davier, Phys. Rev. Letters **20**, 952 (1968).

[14] W. G. Jones, D. Kreinick, R. Anderson, D. Gustavson, J. Johnson, D. Ritson, F. Murphy, M. Gettner, and R. Weinstein, Phys. Rev. Letters **21**, 586 (1968). References for data at photon energies below 6 GeV can be found in this letter.

[15] M. Davier, I. Derado, D. Drickey, D. Fries, R. Mozley, A. Odian, F. Villa, D. Yount, and R. Zdanis, Phys. Rev. Letters **21**, 841 (1968); and report (unpublished).

[16] G. McClellan, N. Mistry, P. Mostek, H. Ogren, A. Silverman, J. Swartz, R. Talman, K. Gottfried, and A. I. Lebedev, Phys. Rev. Letters **22**, 374 (1969).

[17] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München-Collaboration Phys. Rev. **175**, 1669 (1968).

larger for mass values below the resonance than above the resonance. Although we are hesitant to compare our predictions with data taken at photon energies less than about 15 GeV, we mention that this shift was found at lower energies (between 2.5 and 5.8 GeV) by the DESY Bubble-Chamber Collaboration.[17]

In order to study the $m^2$ dependence of the cross section, we abandon the narrow-width approximation in going from Eqs. (34) to (35). Then the $\rho$ contribution to the cross section is given by

$$\frac{d^2\sigma}{dtdm^2}=\frac{\alpha}{3\pi}\beta(m)^3\left(\frac{m_\rho{}^2}{m}\right)^2[(m_\rho{}^2-m^2)^2+m_\rho{}^2\Gamma^2g(m)^2]^{-1}$$
$$\times H(y)e^{at}(64\pi)^{-1}[\sigma_T(\pi^+p)+\sigma_T(\pi^-p)]^2. \quad (38)$$

We have neglected the "background" and "interference" terms in Eq. (34), since they give a relatively small correction over the mass range of interest (the interference term vanishes after integration over the pion angular distribution). The appearance of the *second* power of $m_\rho/m$ is to be contrasted with the *fourth*-power behavior claimed by Ross and Stodolsky [see Eq. (18)]. There is also a rather weak (for small $t$) $m^2$ dependence in the factor $H(y)$.

In Fig. 6 we show the comparison of $d\sigma/dm$ as predicted from Eq. (38) with the 8–16-GeV data of Ref. 15. The comparison might be thought a trifle unfair since the experimental points are integrated over all momentum transfers, whereas the prediction is made for $t=0$. We argue, however, that most of the contribution to the data must be from the small $t$ region because of the rapid falloff of the cross section away from $t=0$, thereby justifying the comparison. In plotting the curve in Fig. (6), an "eyeball" normalization to the data has been made. We consider that the data provide a rather impressive qualitative confirmation of the asymmetric resonance peak predicted by Eq. (38). The data do not, however, appear to be adequate to distinguish between the second- and fourth-power dependence of the $m_\rho/m$ factor.

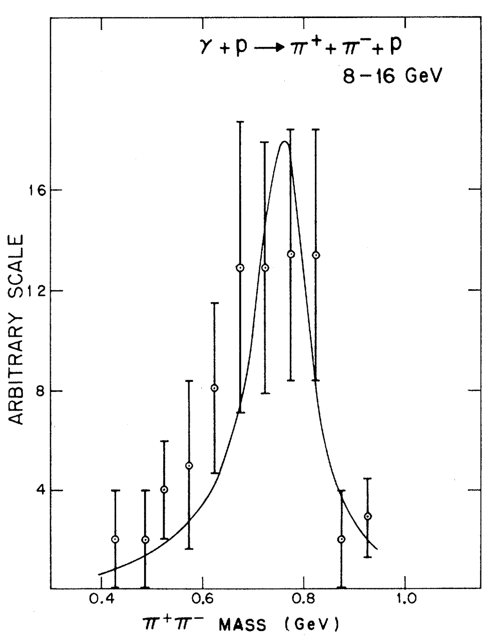


FIG. 6. Dipion mass distribution. The solid curve is the prediction of the theory, normalized to the data by a visual fit. The experimental data are from Ref. 15.

The parameters used in calculating the curve in Fig. 5 were essentially those of Gounaris and Sakurai,[12] namely, $m_\rho=775$ MeV and $\Gamma_\rho=130$ MeV. Just as in the process $e^++e^-\to\rho\to\pi^++\pi^-$, we find a downward mass-shift from $m_\rho$ of about 15 MeV for the peak of the mass distribution.

As stated in the introduction, the model also predicts the density matrix for the $\pi^+\pi^-$ decay angular distribution. In terms of the quantity $y=-t/m^2$ we find that (in the Jackson[18] frame)

$$\rho_{00}=\tfrac{1}{2}y(1+y-2f)^2D^{-1}(y), \quad (39a)$$

$$\rho_{1\,-1}=y[1+2y(f-1)]D^{-1}(y), \quad (39b)$$

$$\mathrm{Re}\rho_{10}=8^{-1/2}y^{1/2}(1+y-2f)(1-3y+2yf)D^{-1}(y), \quad (39c)$$

where

$$D(y)=[1+2y(f-1)]^2+y^2+\tfrac{1}{2}y(1+y-2f)^2 \quad (39d)$$

and $f$ is defined by Eq. (28). For $\beta$ calculated at the $\rho$ mass

$$f=1.33, \quad (39e)$$

and it goes to unity for vanishing $\beta$.

We have remarked earlier that ours is an asymptotic, small-$t$ theory. Consequently, comparison of the predictions of Eq. (39) with experiment should await data obtained in the 15+ GeV region, suitably restricted to small values of $t$.

### B. $\phi$ Photoproduction

As we mentioned in the introduction, the model worked out in Secs. III and IV can be also considered as a model for $\phi$ photoproduction through intermediate $K^+K^-$ pairs. Besides trivial changes such as replacing $m_\rho\to m_\phi$, $\Gamma_\rho\to\Gamma_\phi$, $\sigma_T(\pi^\pm p)\to\sigma_T(K^\pm p)$ and interpreting $m$ as the mass of the final $K^+K^-$ system, we have to modify the final-state interaction factor $F(m^2)$ in Eq. (32). Instead of Eq. (32) we write for $F(m)$ in the vicinity of the $\phi$ resonance

$$F(m)=x(m)m_\phi{}^2[m_\phi{}^2-m^2-im\phi\Gamma_\phi g(m)]^{-1}, \quad (40)$$

with

$$g(m)\equiv[\beta(m)/\beta(m_\phi)]^3(m/m_\phi)^2. \quad (41)$$

[18] The $\rho$ rest system with $z$ axis along the incoming photon beam. See K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964). For density matrices in other models see, for example, G. Kramer and K. Schilling, Z. Physik **191**, 51 (1966); G. Kramer, DESY Report No. 67/32 (unpublished).

In accordance with the discussion in Sec. IV the unknown function $x(m)$ is determined from the process $e^+ + e^- \to \phi \to K^+ + K^-$ by the relation

$$\sigma(e^+ + e^- \to \phi) = \tfrac{1}{3}\pi\alpha^3 m_\phi^{-2}\beta^3 |F(m_\phi)|^2$$
$$= 12\pi m_\phi^{-2}\Gamma_{\phi\to e^+e^-}/\Gamma_{\phi\to \mathrm{all}}. \quad (42)$$

A recent compilation of the measurements on $\phi \to e^+e^-$ reports for the branching ratio[1] that $\Gamma_{\phi\to e^+e^-}/\Gamma_{\phi\to \mathrm{all}} = (3.55+0.48)\times 10^{-4}$. This number yields[19] $x^2(m_\phi) = 0.178$, using for the total width $\Gamma_{\phi\to\mathrm{all}} = (3.4\pm 0.8)$ MeV.[20] To obtain the $\phi$-production cross section, we have to multiply Eq. (35) by $x^2(m_\phi)$. The final formula for $\gamma p \to \phi p$ in the narrow-width approximation, then, is

$$\frac{d\sigma}{dt}(\gamma p \to \phi p) = 4\alpha \frac{4\pi}{g_{\phi\bar{K}K}^2} x^2(m_\phi) \times \frac{[\sigma_T(K^+p) + \sigma_T(K^-p)]^2}{64\pi} H(y_\phi) e^{at}. \quad (43)$$

In this formula, $g_{\phi\bar{K}K}$ is determined from the total width of the $\phi$ assuming that it is all $\bar{K}K$ decay and $a$ is the slope of elastic $K^\pm p$ scattering. Using $\Gamma_\phi = 3.4$ MeV, $\sigma_T(K^\pm p) = 16.4$ mb for the asymptotic $K^\pm p$ total cross section[13] and $a = 8$ GeV$^{-2}$, we obtain for $(d\sigma/dt)(\gamma p \to \phi p)$ in the "forward direction" ($t=0$) the value 24.4 $\mu$b GeV$^{-2}$ and the $t$ distribution shown in Fig. 7. This distribution is compared with the recently published data of Jones *et al.*,[14] for $\gamma$ energies of 13 and 16 GeV. Also here we can say that the comparison is satisfactory for values of $-t$ less than about 0.5 GeV$^2$.

The density matrix for the $K^+K^-$ decay angular distribution is again given by Eqs. (39a)–(39d).

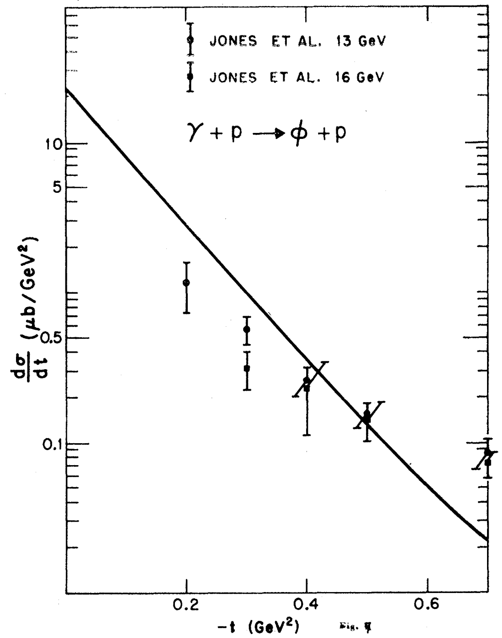


FIG. 7. Differential cross section for $\phi$-meson photoproduction as a function of $t$. The solid curve is the absolute prediction of the theory. The experimental data are from Ref. 15.

## VII. CONCLUSIONS

In this paper we have presented a very simple-minded theory of pion and kaon pair photoproduction at high energies and small momentum transfers. To the extent that the pions or kaons, when they are in a relative $p$ state, undergo final-state interaction, the theory may be considered to be one of $\rho^0$ or $\phi$ photoproduction. However, as is evident from the development, the theory predicts the presence of all odd angular momentum states (away from $t=0$) in a well-defined way. Thus, we might expect to calculate the high-energy photoproduction of a $J^P = 3^-$ resonance (at 1650 GeV?)[20] from the same theory. Nevertheless, the $p$ states are distinguished in that only they survive in the $t=0$ limit.[21]

We have already noted that the prediction for $(d\sigma/dt)(\gamma + p \to \rho^0 + p)$ at $t=0$ from our two-pion theory stands in rather amusing contrast to the corresponding prediction made on the basis of vector dominance plus quark model. Since the two-pion intermediate state represents the scattering of a $qq\bar{q}\bar{q}$ system from the proton (rather than $q\bar{q}$), we predict exactly four times the vector-dominance cross section. Whether or not the factor of 4 represents any real difference in the physical content of the two theories (regarding the existence of quarks) is a moot point. We would prefer to take the viewpoint that the difference really represents different approximations in the treatment of the off-the-mass-shell effects. It should be kept in mind that the quark model is not a necessary part of the vector-dominance philosophy. However, without the quark model the $\rho^0$-nucleon cross section becomes an empirical quantity to be determined from experiment. We, on the other hand, make a definite prediction free of arbitrary parameters.

It does appear, thus far, that the prediction bears a reasonable resemblance to physical reality. In addition, there are other predictions from the model that appears to be in accord with the data. These are the shape of the $\rho$ peak and, qualitatively (at lower energies), the change in the slope of $d\sigma/dt$ as a function of the dipion mass. Predicted, but not tested, are the $\rho^0$ and $\phi$ (or, more precisely, the $\pi^+ - \pi^-$ and $K^+$-$K^-$ $p$-wave) interference with background, the decay density matrix, and pion (kaon) pair production from nuclei. These we leave to the experimentalists.

[19] This agrees with symmetry expectations which yield $x^2(m_\phi) = \frac{1}{9}$.

[20] A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, Matts Roos, Paul Söding, W. J. Willis, C. G. Wahl, Rev. Mod. Phys. **40**, 77 (1968).

[21] This is to be contrasted with some currently popular lore concerning diffraction dissociation by Pomeranchuk exchange, where it is supposed that the quantum numbers of any orbital angular momentum ($0^+$, $1^-$, $2^+ \cdots$) may be exchanged at $t=0$. For a specific model see R. C. Arnold, Phys. Rev. **157**, 1292 (1967).

In our opinion there is still room for considerable improvement in the present model. Our treatment of the gauge-invariance problem was somewhat cavalier, to say the least. The treatment of off-the-mass-shell effects could be sharpened somewhat although we have no clear ideas on the scattering theory of virtual pions (or kaon) from nucleons. The general topic of final-state interactions needs further study. Finally, we have considered only Pomeranchukon exchange in the $\pi-N$ and $K-N$ scattering amplitude. It would be of interest to consider also the contributions of lower trajectories, especially the $P'$ and $\rho$. All of these we leave to later theoretical endeavors.

*Note added in manuscript.* An interesting question has been raised as to the possible relationship between our model and an earlier one due to Berman and Drell.[22]

The prediction of the model of Berman and Drell is that the forward $\rho^0$ production amplitude at high energy is given by

$$\frac{d\sigma}{dt}(\gamma+p\to\rho_0+p)\Big|_{t=0}\approx\frac{1}{16}\left(\frac{g_{\rho\pi\omega}^2}{4\pi}\right)\left(\frac{g_{\gamma\pi\omega}^2}{4\pi}\right)\left(\frac{4\pi}{g_{\rho\pi\pi}^2}\right)^2\frac{d\sigma_{\pi N}}{dt}\Big|_{t=0},\quad(44)$$

where the $\pi-N$ cross section is the average of $\pi^+p$ and $\pi^-p$.

It is instructive to rewrite Eq. (44) using the relations

$$g_{\gamma\pi\omega}=(e/2\gamma_\rho)g_{\rho\pi\omega}=e(g_{\rho\pi\omega}/g_{\rho\pi\pi})\quad(45)$$

and

$$(g_{\rho\pi\omega}/g_{\rho\pi\pi})^2\approx4.\quad(46)$$

The first relation comes from assuming universal coupling of vector mesons[22] and vector dominance of the pion form factors. The second is an estimate[23] based upon $SU(6)$ and gives the $\omega$ width within a factor of 2 in the decay model of Gell-Mann, Sharp, and Wagner.[24] With these substitutions, we find that Eq. (44) becomes

$$\frac{d\sigma}{dt}(\gamma+p\to\rho_0+p)\Big|_{t=0}\approx\alpha\left(\frac{4\pi}{g_{\rho\pi\pi}^2}\right)\frac{d\sigma_{\pi N}}{dt}\Big|_{t=0},\quad(47)$$

which is just the vector dominance plus quark-model prediction. On the other hand, if $g_{\rho\pi\omega}$ is determined from the experimentally known $\omega$ width by the method of Gell-Mann, Sharp, and Wagner, then the value given by Eq. (46) is too small by a factor of about 2 and the cross section in Eq. (47) should be increased by a factor of 4, thereby giving just the prediction of our model.

Can it be, then, that the two models are describing the same mechanism? We would suggest that they very well might be on the basis of the following argument. Consider the $\omega$ exchanged at the top rung of the ladder in Berman and Drell's Fig. 3. Decompose the $\omega$ into a real pion constituting the top rung of the ladder and a virtual $\rho$ crossing to make the second rung, the two being joined by vertical pion lines and the photon and final $\rho^0$ each "hooking" onto the $\pi$-$\pi$ corner. The part of the graph below the top rung then represents pion-nucleon diffraction scattering as in Berman and Drell's Fig. 2(b) and the entire graph is a representation of our Fig. 3(b).

We regret having omitted Ref. 22 from a report of this paper.

**ACKNOWLEDGMENTS**

We are deeply grateful to David Horn for his involvement in an early phase of this calculation and for his contribution of a philosophy concerning the treatment of final-state interactions. We also wish to thank the authors of Refs. 14–17 for reports of their work. One of us (G.K.) thanks the Argonne High Energy Physics Theory Group for the kind hospitality extended to him and for providing a stimulating atmosphere.

22 S. M. Berman and S. D. Drell, Phys. Rev. **133B**, 791 (1964).
23 B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965).
24 M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).